Math 441
Quiz 1
8/30/13

Name Key

Show All Work

1) Set up a system of equations that would allow you to determine a polynomial whose graph passes through (-1, 4), (0, 2), (1, 3), (2, 1).

$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$\begin{aligned} a_0 - a_1 + a_2 - a_3 &= 4 \\ a_0 &= 2 \\ a_0 + a_1 + a_2 + a_3 &= 3 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 4 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 8 & 1 \end{array}\right]$$

2) Solve the following system using either Gaussian or Gauss Jordan elimination on the augmented matrices.

a)
$$\begin{aligned} x + 2y + 6z &= 1 \\ 2x + 5y + 15z &= 4 \\ 3x + y + 3z &= -6 \end{aligned}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 6 & 1 \\ 2 & 5 & 15 & 4 \\ 3 & 1 & 3 & -6 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 1 \\ 0 & 1 & 3 & 2 \\ 3 & 1 & 3 & -6 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_3} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -5 & -15 & -9 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 2 \\ 0 & -5 & -15 & -9 \end{array}\right]$$

$$\xrightarrow{5R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right] \leftarrow \text{NO SOLUTION}$$

b) $\begin{aligned} 2x + 4y - 7z &= 0 \\ x - 3y + 9z &= 0 \\ 3x - 3y + 12z &= 0 \end{aligned}$ $\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 9 & 0 \\ 2 & 4 & -7 & 0 \\ 3 & -3 & 12 & 0 \end{array}\right]$ Switched Rows as copied

$\xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -3 & 9 & 0 \\ 0 & 10 & -25 & 0 \\ 0 & 6 & -15 & 0 \end{array}\right] \xrightarrow{\frac{1}{10}R_2} \left[\begin{array}{ccc|c} 1 & -3 & 9 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 6 & -15 & 0 \end{array}\right]$

$\xrightarrow[-6R_2+R_3]{3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + \frac{3}{2}x_3 = 0$

$x_2 - \frac{5}{2}x_3 = 0$

let $x_3 = t$

$$\left\{ \left(-\frac{3}{2}t, \frac{5}{2}t, t \right) \mid t \in \mathbb{R} \right\}$$